On Thinking About & Doing Mathematics

by Don Cohen

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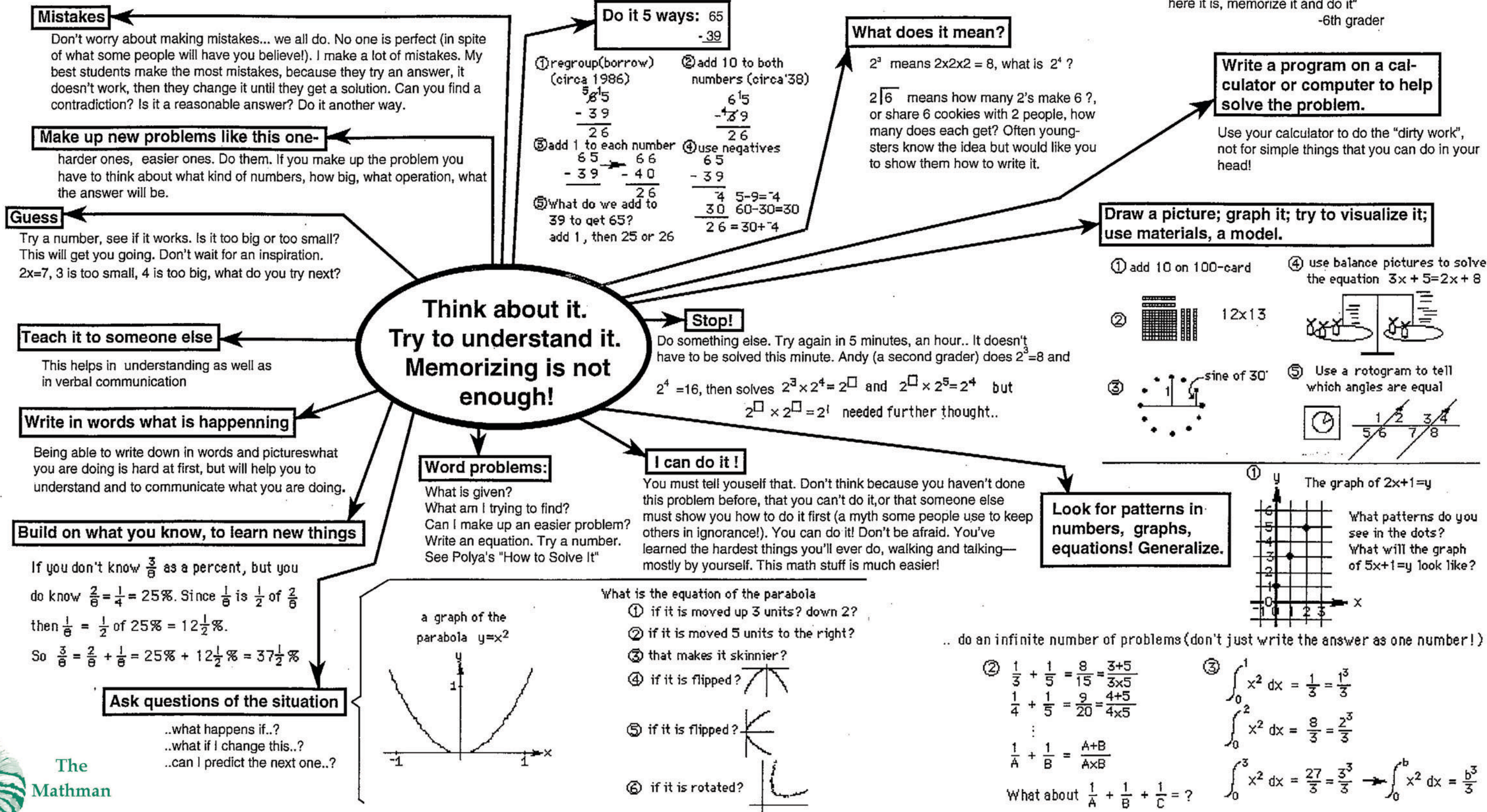
Stay with it, don't give up, and have fun!

Think about it. Try to understand it. Memorizing is not enough!

Mistakes

Don't worry about making mistakes... we all do. No one is perfect (in spite of what some people will have you believe!). I make a lot of mistakes. My best students make the most mistakes, because they try an answer, it doesn't work, then they change it until they get a solution. Can you find a contradiction? Is it a reasonable answer? Do it another way.

Make up new problems like this one-

harder ones, easier ones. Do them. If you make up the problem you have to think about what kind of numbers, how big, what operation, what the answer will be.

Guess

Try a number, see if it works. Is it too big or too small? This will get you going. Don't wait for an inspiration. $2x=7$, 3 is too small, 4 is too big, what do you try next?

Teach it to someone else

This helps in understanding as well as in verbal communication

Write in words what is happenning

Being able to write down in words and pictures what you are doing is hard at first, but will help you to understand and to communicate what you are doing.

Build on what you know, to learn new things

If you don't know $\frac{3}{8}$ as a percent, but you do know $\frac{2}{8} = \frac{1}{4} = 25\%$. Since $\frac{1}{8}$ is $\frac{1}{2}$ of $\frac{2}{8}$ then $\frac{1}{8} = \frac{1}{2}$ of $25\% = 12\frac{1}{2}\%$. So $\frac{3}{8} = \frac{2}{8} + \frac{1}{8} = 25\% + 12\frac{1}{2}\% = 37\frac{1}{2}\%$

Ask questions of the situation

..what happens if..?
..what if I change this..?
..can I predict the next one..?

Do it 5 ways: $\begin{array}{r} 65 \\ -39 \end{array}$

① regroup(borrow) (circa 1986)

② add 10 to both numbers (circa '38)

③ add 1 to each number
$\begin{array}{r} 65 \\ -39 \\ \hline \end{array}$ $\begin{array}{r} 66 \\ -40 \\ \hline 26 \end{array}$

④ use negatives
$\begin{array}{r} 65 \\ -39 \\ \hline \end{array}$
$\begin{array}{r} -4 \\ 30 \\ \hline 26 \end{array}$ $\begin{array}{l} 5-9=-4 \\ 60-30=30 \\ 26=30+-4 \end{array}$

⑤ What do we add to 39 to get 65? add 1, then 25 or 26

What does it mean?

2^3 means $2 \times 2 \times 2 = 8$, what is 2^4 ?

$2\overline{)6}$ means how many 2's make 6 ?, or share 6 cookies with 2 people, how many does each get? Often youngsters know the idea but would like you to show them how to write it.

Word problems:

What is given?
What am I trying to find?
Can I make up an easier problem?
Write an equation. Try a number.
See Polya's "How to Solve It"

I can do it !

You must tell youself that. Don't think because you haven't done this problem before, that you can't do it, or that someone else must show you how to do it first (a myth some people use to keep others in ignorance!). You can do it! Don't be afraid. You've learned the hardest things you'll ever do, walking and talking— mostly by yourself. This math stuff is much easier!

Stop!

Do something else. Try again in 5 minutes, an hour.. It doesn't have to be solved this minute. Andy (a second grader) does $2^3=8$ and $2^4=16$, then solves $2^3 \times 2^4 = 2^\square$ and $2^\square \times 2^5 = 2^4$ but $2^\square \times 2^\square = 2^1$ needed further thought..

Write a program on a calculator or computer to help solve the problem.

Use your calculator to do the "dirty work", not for simple things that you can do in your head!

Draw a picture; graph it; try to visualize it; use materials, a model.

① add 10 on 100-card

② 12x13

③ sine of 30

④ use balance pictures to solve the equation $3x + 5 = 2x + 8$

⑤ Use a rotogram to tell which angles are equal

$\frac{1}{5} \frac{2}{6} \frac{3}{7} \frac{4}{8}$

Look for patterns in numbers, graphs, equations! Generalize.

① The graph of $2x+1=y$

What patterns do you see in the dots? What will the graph of $5x+1=y$ look like?

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a graph of the parabola $y=x^2$

What is the equation of the parabola
① if it is moved up 3 units? down 2?
② if it is moved 5 units to the right?
③ that makes it skinnier?
④ if it is flipped?
⑤ if it is flipped?
⑥ if it is rotated?

.. do an infinite number of problems (don't just write the answer as one number!)

② $\frac{1}{3} + \frac{1}{5} = \frac{8}{15} = \frac{3+5}{3 \times 5}$
$\frac{1}{4} + \frac{1}{5} = \frac{9}{20} = \frac{4+5}{4 \times 5}$
⋮
$\frac{1}{A} + \frac{1}{B} = \frac{A+B}{A \times B}$
What about $\frac{1}{A} + \frac{1}{B} + \frac{1}{C} = ?$

③ $\int_0^1 x^2 \, dx = \frac{1}{3} = \frac{1^3}{3}$
$\int_0^2 x^2 \, dx = \frac{8}{3} = \frac{2^3}{3}$
$\int_0^3 x^2 \, dx = \frac{27}{3} = \frac{3^3}{3} \rightarrow \int_0^b x^2 \, dx = \frac{b^3}{3}$

Learning will never be finished.. there will always be new questions, new problems and always, much confusion .. so enjoy !

The Mathman